Synopsis of
Amity and Enmity
Vol I
by
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Introduction

Amity and Enmity: Two Archetypes of Social Existence, An Interdisciplinary Study, 2003, Editions à la Carte, Züri, is a three volume study written by Dr. Rudolf Starkermann, a retired professor of mechanical engineering. Dr. Starkermann applies his expertise in the area of control system to model socio-psychological behavior. These models have simple relations based on goals, will power, conscious and unconscious transfers of information, and the resultant behaviors. The key finding and exploration in the first volume is that of the three possible relationships in a dualism; autonomous, consensual, and hostile, hostility has far greater stability and speed over a consensual partnership. Another way to state this is that nature favors aggression. Starkermann’s ideas have been influential in the systems engineering world. Many of his concepts, including those from Amity and Enmity, were fundamental elements in the 2007 INCOSE Intelligent Enterprises Working Group report.

This synopsis of Volume I is intended to provide a concise overview to the systems engineering audience. An ability to follow control diagrams is helpful. Starkermann lives in Switzerland and suffers being relatively unknown in the United States. This is due in part to the difficulty in finding a translator familiar with the material able to take it from the original German text. The English translation has some awkward phrases and terms and many times the non-English proverbs he uses to make a point are not translated. Starkermann takes care to communicate thoroughly to both of his audiences: engineering and social behavior. Because of this much time is spent explaining concepts twice; once for each paradigm. This exercise has a similar obligation – Starkermann’s examples, charts, and appropriate use of proverbs and sayings will be used as much as possible so familiarity with reading graphs is required. Even so, this treatment will not attempt go into great depth with formulae or numerical extrapolations. The reader is encouraged to explore the source work for that information.

The text of Vol I covers ten topics. In Topics IV and V the author expands the discussion through subchapters. The net is nineteen chapters which this paper will attempt to summarize in seven sections. The word will when used as a noun has an important and specific meaning in this work and shall be italicized. Italics will not be used when the word is as a verb. All charts and diagrams with the exception of Figures 1 and 10 through 13 have been scanned in from the original text.

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1 - Natural Law and the Individual model

Reflecting Reality
The processes of the universe are subject to the natural laws of physics. It follows that these laws are extensible in some manner to psychological and social behaviors. This assertion that socio-psychological behaviors can be described mathematically has met with resistance from both humanities and natural sciences. The difficulty of quantifying this behavior through modeling and simulation is acknowledged and it is obvious any models will increase in complexity quickly. Where the humanities deal with qualitative descriptions, this approach is not sufficient for scientific work as we see in the following:

In the example of throwing a stone in the air the qualitative description is:

“If a stone is thrown into the air, it goes, makes a kind of an arch, and comes back to the ground. The harder it is thrown the higher up or the further it goes.”

This does not provide any definitive information about the height or distance of the throw or the true shape of the arc. This is given in the quantitative description given in Equation 1:

\[
h = s \cdot \tan \beta - \frac{g}{2} \cdot \frac{1}{v_0^2 \cos^2 \beta} s^2
\]  

\[ (1) \]

Where: \( h \) = height, \( \beta \) = angle from the ground, \( v_0 \) = starting velocity, \( g \) = gravitational constant, \( s \) = distance the stone is thrown.

From this formula a trajectory can be calculated and charted as shown to the right.

The point here is that quantitative descriptions can give meaning to the qualitative. This is the rational for developing measurable models to extend understanding in the social realm. It is the duty of the scientist to make a model that reflects reality.

![Figure 1 - Ball trajectory](image-url)
The Social Unit

Social connections consist of at least two parties. Starkermann uses the terms P₁ and P₂ to define the partners in his models. P₁ and P₂ develop interactions based on behavior as well as perception or attitudes towards each other.

Cooperation and hostility between P₁ and P₂ can be described in qualitative contexts. Cooperative parties help each other and increase their self-realization (i.e. goal achievements). Amicable behavior provides mutual help. Opposing that, hostility decreases the self-realization for each partner. Hostility causes mutual damage. The third description for interaction is when P₁ and P₂ are autonomous from each other. This is the same as an individual model. It indicates indifference or neutrality between parties.

Analysis of the consensual and hostile interactions leads to interesting results. The intensity of both consent and hostility can vary. However, the effects from the intensity are different. Mutual harm from hostility will be greater than the help gained from mutual consent for a given level of intensity.

The speed of action (v) of P₁ and P₂ is also affected by their relationships. The speed of acting autonomously considered the same for both parties. In a hostile relation the speed is found to be almost as fast as the autonomous speed. The speed in a consensual relation is much slower than the hostile one. In other words, enmity moves faster than amity.

Whether P₁ and P₂ are acting with amity or enmity to each other, the interlinking of attitudes and behaviors forms a constantly adjusting cause-effect loop between them. Cause generates effect and the effect in turn generates a cause. Even a simple model with few interactions requires iterative simulation for a rational result.

The element missing from these descriptions is a quantitative measure of how they interoperate. A model that can measurably map out these interactions has value.

The model of the individual provides the fundamental unit necessary for this study and allows for more complex constructions. P₁ is the reference given this model. Starkermann’s findings for the nature of the individual are:

1) Self-realization is an individual’s predominant goal.
2) An individual’s will directly impacts his drive to achieve goals and this in turn affects the attainment of goals.
3) The more an action is procrastinated the less will that can be exerted for goal attainment.
4) The less the will of an individual, the more a disturbance can affect him either positively or negatively.
5) Excessive will drives an individual past the point of stability, and, in Starkermann’s terms past the point of self-realization.
6) Swiftness of action and synergism with will define a unit’s intelligence.
Three postulates (paraphrased for brevity) frame the development of the models.

Postulate 1: Social behavioral characteristics predicted by any mathematical model must reflect real world observations of regular behavior.

Postulate 2: The qualities of social systems are evaluated on the basis of comparison. There are no measurable standards or values except for the differences between units.

Postulate 3: Social behavior is incredibly complex such that models can only be built with restricted assumptions and simplifications.

Figure 2 represents the basic model for the individual \( P_1 \). It is constructed using standard elements and notations from control systems. Conversely, the interpretations of the effects of the model’s results draw from sociology and psychology.

The blocks represent Laplace transforms and time domain values in place of differential equations in the frequency domain. However there is no requirement in this volume of Starkermann’s works to perform Laplace transforms; merely to understand what they represent. As signals or data move through a block, the block essentially multiplies its function across the signal. Summing blocks add signals.

The block elements are:
Goals \( (u_{11}) \), Outside disturbances \( (u_{12}, u_{13}) \), Will \( (G_1) \), Self \( (S_{11}) \), Attitude exchange \( (S_{12}, S_{21}) \), Functional operations \( (F_1) \), Error signal \( (\varepsilon_1) \), Observational interaction signals \( (V_{12}, V_{21}) \), Physical interaction signals \( (A_{12}, A_{21}) \), Behavior, i.e. goal attainment actions \( (x_1) \), Feedback factor, perception of attainment \( (R_1) \), Transfer of \( x_1 \) goal attainment to public \( (C_1) \).

The section labeled “CN” represents the conscious elements of the individual’s behavior. The section labeled “UC” is the unconscious part.

The numeric indices are in the standard form for controls and indicate direction of a signal. “12” means to “1” from “2” and “21” means to “2” from “1”. \( S_{11} \) is a special case for the individual’s unconscious transfer signal in relation to itself.

\( R_1 \) is generally given the value of -1 in order to provide a negative feedback to compare \( x_1 \) to \( u_{11} \).
The $F_1$ block consists of a time factor and three first order differential operators in series to indicate reaction time for $P_1$. This time delay between the application of Will ($G_1$) toward level of attainment ($x_1$) creates the time dependence of the fundamental model and the need to perform iterative loops on the analysis. All input and output factors and variables $\delta_1, \varepsilon_1, x_1, y_1, \delta_2$, and $\varepsilon_2$ are also time dependent.

**Analogy to a mechanical control model**

Starkermann proceeds to show how there is a valid unconscious section in his model by comparing it to a room thermostat control system. This is a slightly simpler representation than the model for $P_1$ in that factors $A_{12}, A_{21}, V_{12}, V_{21}, S_{12}$, and $S_{21}$ are not present.

The diagram works as follows:

a) a temperature is set on a thermostat ($u_{11}$)  
b) the difference between $u_{11}$ and room temperature is taken ($\Sigma_1$)  
c) the error ($\varepsilon_1$) is used to turn on heating/cooling ($G_1$)  
d) disturbance from the environment are combined with the output of $G_1$  
e) The room ($S_{11}$) is acted upon.  
f) The temperature is read and compared and then acted upon to repeat the loop.

The environment of the room in this diagram is represented by $S_{11}$. To Starkermann the room corresponds to the unconscious $S_{11}$ from the previous model. The room is the transformative environment for temperature change. In the same way the unconscious is the transformative boundary for goal attainment and self.

**2 - Characteristics of the Individual**

There are three main characteristics of the individual model that define its limits and fundamental behavior. Application of Starkermann’s ideas requires understanding these concepts.

The results of action toward achieving goal $u_{11}$.
The remaining effect of disturbance $u_{12}$.
The stable areas of action - which will be in homeostasis.
**Action towards achieving \( u_{11} \)**

The transfer function for goal attainment \( (x_1) \) relative to the goal \( (u_{11}) \) as a function of will \( G_1 \) is shown Equation 2. This is also called self-realization. Analyzing the system in an idealized steady state with \( S_{11} \) and \( F_1 = 1 \), and all disturbances and other inputs = 0 generates the chart shown in Figure 4.

\[
\frac{x_1}{u_{11}} = \frac{G_1}{1 + G_1}
\]  

(2)

The graph shows goal achievement of 100% requires \( G_1 = \infty \). After the 89% level where \( G_1 = 8 \), \( G_1 \) must increase drastically for relatively little gain. Also to note in this basic model 50% goal achievement is reached with \( G_1 = 1 \) and 80% is reached with \( G_1 = 4 \).

While this is a steady state it is not a static state. The goal attainment is based on the close-system feedback loop. It shows that tremendous will is required for total goal attainment with diminishing returns. What is not yet shown is that too large a will can cause the system to become unstable.

**Effect of Disturbance \( u_{12} \) on \( u_{11} \)**

When a disturbance \( u_{12} \) is taken into account, the effect can be either negative or positive with respect to \( u_{11} \). The result is that \( u_{12} \) can help achieve up to 100% of P1’s goal when \( u_{12} \) is positive and equal to \( u_{11} \). When \( u_{12} \) is negative it can reduce goal attainment despite the level of will. The transfer function for \( x_1/u_{12} \) is:
\[
\frac{x_1}{u_{12}} = \frac{1}{1 + G_1}
\]  

(3)

The result of the impact from \( u_{12} \) can be seen in Figure 5. These show only the case for \( u_{12} = u_{11} \). Note for \( u_{12} > 0 \), \( x_1/u_{11} + x_1/u_{12} \leq 0 \) since goal attainment is limited to 100%.

![Figure 5 - Effect of \( u_{12} \) on Goal Attainment](image)

The point to be made here is that the will required for a particular goal attainment level is impacted by the disturbances that enter the system. More will is required to achieve a goal when there is a negative disturbance.

**Stability of the Individual**

The third fundamental concept is that there are limits of stability for the individual and, as will be seen later for any interactions between parties. Time becomes a factor in being able to act and multiple delays and the way they are acted upon can have domains of instability. With time as a factor, the physiological function of memory is implied as an actor as well. The limit of stability is not a fixed state but, using another term from physiology, one of homeostasis – a dynamic equilibrium. The basic homeostatic range for our individual \( P_1 \) is generated from the transfer functions in block \( F_1 \) of the model.

\( F_1 \) uses the Laplace transform from Equation 4.

\[
\frac{1}{(T_1s + 1)^m}
\]

(4)
This is a time delay factor of $T$ multiplied $m$ times. While each delay stage can have a unique $T$, they are kept equal at this point to simplify analysis and explanation. Each instance of the function represents 1 delay unit. Starkermann’s model insists on a minimum of three time delays ($m=3$) since that is the minimum that number that can create instabilities. And as he points out, people do become unstable.

The function of the time delays can be illustrated by a series of water tanks that flow from one to another. For the purpose of this example the tanks in Figure 6 are considered identical. In a steady state situation the flow into the system equals the flow out of and the water levels in each tank stay constant.

In the case when the inlet flow increases the first tank will establish a new level. It takes an amount of time for the levels of all the tanks to become equal again. Each following level adjusts more slowly than the one before it. This is related to the height, $H$ of the tank and the amount of change $\Delta H$. In one type of unstable situation the inlet flow becomes too great and the tanks will overflow.

The delay in $F_1$ relates to the ability to act in relation to the goal as expressed through the will $G_1$. This can be a decision making process, the time to drive to a store, waiting in a line – anything a person does. Starkermann requires at least three delay stages in his social model. Three is the minimum number of differential delays that can produce an unstable system. And human behavior is fraught with instability, fluctuations, and uncertainty. As a person has more internal delays, will is reduced for application to goals.

The maximum amount of will that can be applied through $F_1$ is directly related to the order $m$ of $F_1$. For the case of $T_1=T_2=T_3$, an $F_1$ with $m=3$ allows a maximum will where $G_1 = 8$. For $m = 4$, $G_1 \approx 4$. When the ratio of $T_n$ for the delays $\neq 1$, $G_1$ increases. For $T_1/T_2=1$ and $T_1/T_3=10$ then $G_1 \approx 25$. Different $T$’s relate to the different kinds of internal decision and acting sequences a person may have. While the ratio of $T_n$ can increase $G_1$, the velocity of action depends on the aggregate value of the $T$’s. Velocity refers to the rate of oscillation of the system when it is in marginal stability.

![Figure 6 - Waterfall example](image-url)
Starkermann relates this speed of action for $F_1$ to the intelligence of the system. That is, the faster and closer a system can attain its goal defines intelligence. This is the combination of speed and will. Table 1 illustrates how these factors relate for four different $T_n$ units and their ratios.

<table>
<thead>
<tr>
<th>Unit</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$G_1$</th>
<th>$v$</th>
<th>$x_1/u_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>1</td>
<td>89%</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>0.1</td>
<td>24.2</td>
<td>2.7</td>
<td>96%</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>24.2</td>
<td>6.3</td>
<td>96% Accurate</td>
</tr>
<tr>
<td>D</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>8</td>
<td>10</td>
<td>89% Fast</td>
</tr>
</tbody>
</table>

In Table 1 the question of which unit is superior comes down to a choice between C and D. C with a much higher will is superior if goal attainment is the critical criteria. Unit D is superior if speed is more important. Life reflects this – speed and power are critical to success.

At this point this most basic autonomous model consists of will, goal attainment, model flow, disturbances, and feedback adding to its complexity. Yet it is still very simplistic when viewed from a social standpoint.
3 - Dualism

We have an initial model for the interaction of $P_1$ and $P_2$. At present it only shows the unconscious interaction. The unconscious elements (UC) form the "Archaic surviving package". The self-realization of $P_1$ and $P_2$ depend wholly on whether they are in a state of amity or enmity. The terms $S_{12}$ and $S_{21}$ reflect the unconscious attitude of one party to the other.

Amity exists when the unconscious attitude of one party is devotional or altruistic for their partner and the other is egoistic. The devotional partner has to accept the opinion of the other. The relationship of Amity is represented by $S_{12}S_{21} = -1$. Enmity exists when $S_{21}S_{12} = 1$. Neither partner subsumes their opinion to the other.

For this discussion $S_{21}S_{12}$ only equals 1 when both $S_{12}=1$ and $S_{21}=1$. The special case of $S_{12}=-1$ and $S_{21}=-1$ indicates both parties act devotional to their partner. Starkermann describes this last case as being "sociologically pathological". The behavior of $S_{12} = S_{21} = -1$ is lightly examined later in Vol I and is explored in greater depth in Vol III.

The transfer function representing the dualism can be shown as

$$ (1 + G_1 F_1 S_{11}) + (1 + G_2 F_2 S_{22}) + G_1 G_2 F_1 F_2 S_{12} S_{21} = 0 \quad (5) $$

$P_1$ is represented by:

$$ (1 + G_1 F_1 S_{11}) = 0 \quad (6) $$

$P_2$ is represented by:

$$ (1 + G_2 F_2 S_{22}) = 0 \quad (7) $$

The linking between the partners, also called a coupling loop is:

$$ G_1 G_2 F_1 F_2 S_{12} S_{21} = 0 \quad (8) $$
The coupling factor, i.e. the product of $S_{12}S_{21}$, shows how the partners are bound to each other by their unconscious attitudes. At this point it is important to explain how $S_{12}S_{21} = -1$ as amity and $S_{12}S_{21} = 1$ as enmity work.

Assuming $S_{12} = -1$ we arrive at values for $x_1$ and $x_2$ as:

$$x_1 = S_{11}\delta_1 - S_{12}\delta_2$$  \hspace{1cm} (9)

$$x_2 = S_{22}\delta_2 + S_{21}\delta_1$$  \hspace{1cm} (10)

$P_1$’s internal error is reduced by $S_{12}\delta_2$; here Starkermann uses the term “feedcross signal”. This feedcross signal gives an unconscious indication of being closer to achieving $P_1$’s goal. After passing through $\Sigma_1$, the error $\varepsilon_1$ is larger, giving a conscious view that the goal is farther away. The reverse happens with the other partner. The net effect from the looping process is that self-realization and goal attainment for both increases. Except for the case of parity of wills, one partner must have greater will. He must be aggressive and lead. The other partner with lesser will has to follow. The follower is called the altruist. The leader is the egoist. Even though the system exhibits agreement, it is not mutual in both parties. This cooperation defines friendship.

The feedcross patterns for $S_{12}S_{21} = 1$ are:

$$x_1 = S_{11}\delta_1 + S_{12}\delta_2$$  \hspace{1cm} (11)

$$x_2 = S_{22}\delta_2 + S_{21}\delta_1$$  \hspace{1cm} (12)

By adding the feedcross, the unconscious of both partners feel as though the internal error is greater than what is presented at the conscious level. This disparity causes frustration and is the genesis of anger. Anger applied to the partner is hate. The looping of the model with $S_{12}S_{21} = 1$ drastically and mutually decreases self-realization. Thus we have enmity.

Figure 9 explores the characteristic of goal attainment in a dualism as affected by relationships of amity and enmity. The abscissa is $P_1$’s will $G_1$; the ordinate is $P_1$’s self-realization. The various lines represent the effect of $P_2$’s will when applied in hostility ($S_{12}S_{21} = 1$) and devotion ($S_{12}S_{21} = -1$).
The importance here is that $G_2$ in enmity against $G_1$ can cause tremendous damage. When $G_1=G_2=4$ for enmity, goal attainment is approximately 40%. Hostility makes it very easy to cause tremendous damage to an opponent.

Conversely when $G_2$ is in amity with $G_1$ there is some gain over the autonomous $G_1$ towards goal attainment. The increase in goal attainment is not as great as the damage cause by enmity when all other factors are the same. This shows the power of a hostile relationship towards damaging an opponent’s goals.

**Stability in the Dualism**

As seen for the autonomous unit, the measure of stability must account for time and duration. Stability of the system whether in enmity or amity can be found by building the dualism model in Simulink/Matlab and running it for various levels of for $S_{12}S_{21}=-1$ (amity) and $1$ (enmity). There are particular values of $G_1$ and $G_2$ and where the model develops an oscillation. In control systems this state is called marginal stability. Charting the marginal stability range of $G_1$ vs. $G_2$ creates partially bounded (enmity) and bounded areas (amity). These boundaries of stability have been defined as the **Starkermann resonance**, an apt term to define control system stability in a social context.

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3 Rick Dove, Paradigm Shift International, 17 December 2007. Email correspondence between the author and Mr. Dove where Mr. Dove challenged the author to describe the “Starkermann resonance” of the infamous Stanford Prison experiments of 1971. Hence the term was coined.
It is necessary to graphically demonstrate the system behavior for the terms stability, marginal stability, and instability. The following charts for $x_1$ were created in Simulink (2007a, The MathWorks, Natick, MA) from a Starkermann model in a devotional relationship ($S_{12} = -1$, $S_{21} = 1$). The models were run for a 100 second simulation time. Figure 10 shows the case of stability. The system swings between values until it normalizes at a particular value – approximately 0.52 here. This chart was generated with $G_1 = G_2 = 1.5$. This is well within the stable region for amity shown later in Figure 15.

![Figure 10 – Stability of $x_1$ : $G_1 = G_2 = 1.5$](image)

Marginal stability is demonstrated in Figure 11. For this example $G_1 = G_2 = 2$. The model does not settle to a single value but oscillates at a steady frequency. Its amplitude varies between -0.05 and 1.25. Marginal stability represents the boundary separating stability and instability; order and chaos; sanity and madness.

![Figure 11 - Marginal Stability of $x_1$ : $G_1 = G_2 = 2.0$](image)
Instability is demonstrated in Figure 12. For this example $G_1 = G_2 = 2.2$. The system develops an oscillation that increases the signal amplitude drastically over time. The amplitude reaches -16; much greater than the ranges of the previous two examples.

![Figure 12 – Instability of $x_1$ : $G_1 = G_2 = 2.2$](image1)

When the values for $G_1$ and $G_2$ are both raised from 2.2 to 2.5 the growth of the signal is more drastic. The signal in Figure 13 grow to 1250, almost two orders of magnitude greater than the absolute value for $G_1 = G_2 = 2.2$. Instability can be likened to irrational changing behavior, vacillation of will, or even complete withdrawal and indecision.

![Figure 13 – Instability of $x_1$ : $G_1 = G_2 = 2.5$](image2)
**Stability in Enmity**

The stability of the case of enmity is shown in the chart labeled Hostility. $G_1$ and $G_2$ combine to have a maximum combined willpower of 8. The maximum level for equal wills is $G_1=G_2=4$. The diagonal lines represent constant speed of action. An aspect illustrated here is the stability range extends into infinity. A negative $G_1$ implies an unstable $P_1$ working against his own goals to destroy either a stable or unstable $P_2$. A positive $G_1$ outside the stability range indicates a driven will so strong that the system cannot contain it. Megalomania live in this realm.

**Stability in Amity**

The stability pattern for Amity at $S_{21}S_{12}=-1$ is shown to the right. The striking observation is that the maximum levels for wills that are equal is when $G_1=G_2=2$. The area of stability is much less than for enmity. When a partner in consent tries to increase his will his partner has to keep or reduce his will. If this does not occur, the dualism will not be stable as a friendship but will most likely be stable in the realm of enmity. This illustrates why it is easy to make a friend into an enemy but that the reverse is almost impossible.
Nature’s Prerogative

Parity of willpower in enmity is \( G_1 = G_2 = 4 \) with a velocity of action of 100. In amity willpower parity is \( G_1 = G_2 = 2 \) and velocity is 60. An enemy does not have to rely on cooperation for concurrence from an opponent and can act faster with greater will. The English proverb “A bold attack is half the battle” speaks to this.

Starkermann comments “In nature there is neither peace nor war, there is survival by fight or flight.” Hostility is required by nature for survival and has a simpler structure than a devotional relationship. The stability induced hostility can be examined in the case of \( P_2 \) with \( G_2 = -6 \). This implies instability when autonomous. But when combined with a \( P_1 \) with a stable \( G_2 \), say of 7 or even an unstable \( G_2 \) of 12, the relationship of the two against each other actually induces stability. Two enemies, say countries that are unstable can work against each other and maintain some stability. However in this case \( P_2 \) with its negative will cannot succeed. The conclusion is that hostility is solid and friendship is fragile.

The illustrative point Starkermann makes is that nature is indifferent. Survival relies on having power (via will) and speed. Nature reproduces and adapts very quickly and destruction between parties is inevitable. Hostility is the stable fast medium that nature works through.

Strong and Weak Enmity, Strong and Weak Amity

Until this point the coupling factor \( S_{12}S_{21} \) has been -1 for amity and 1 for enmity. This was done to maintain simplicity of explanation. In reality this factor can vary. \( S_{12}S_{21} = 0 \) is the autonomous state. When either \( S_{12} \) or \( S_{21} \) is 0 it means at least one member does not care. The other party can hate or dislike but there is no reciprocity.

Figure 16 illustrates the case of \( S_{12}S_{21} = +0.0 \ldots +3.0 \). Figure 17 shows amity with \( S_{12}S_{21} = -3 \ldots -0.0 \). Hatred can range from mild dislike to murderous range. Friendships can go from casual to passionate. These charts illustrate the effects of those ranges.

As enmity increases, the area of stability reduces. The extreme result is that in high hostility self-realization becomes less important than harming the enemy.

Figure 16 - Stability for varying hostility
Stability increases as amity lessens and moves toward autonomy. Casual friendships are more stable than strong ones. A particular note is that in a strong friendship, the egoist’s will extends far beyond the limit for autonomous action. But it depends on the altruist’s will to not go into areas of instability. In this situation the weaker willed partner controls the stronger willed one.

For both amity and enmity, will decreases as the unconscious attitude $S_{12}S_{21}$ gains in magnitude.

Starkermann quotes the following observations as they are reflective of these relationships:

- One Englishman is in the golf club,
  Two Englishmen are a golf club,
  Three Englishmen are the Commonwealth.

- One German is a philosopher,
  Two Germans are an organization,
  Three Germans are a war.

- One Frenchman is a poet,
  Two French are an alliance,
  Three French are a marriage.

The interactions discussed are all in the unconscious. The extreme results show that engaging in a hostile or devotional partnership with a partner of much greater will creates strict limits due to the reduced area of stability.

**Self Realization in Extension**

The analysis of the autonomous unit has been focused on self-realization. When dualism was explored the implication was that goals $u_1$ and $u_2$ were incompatible. This, like in life, is not necessarily so.

Goals $u_1$ and $u_2$ can have the following relations:
- They are independent. But one goal will dominate
- They are correlated. However each individual has his own realization
- They are antagonistic. This is mostly seen in hostile relations. For one goal to succeed the other goal must be defeated.
Each goal impacts self-realization differently. So far all observations have revolved around the unconscious coupling in the duality. When the conscious relations $A_n$ (physical interactions) and $V_n$ (observational interactions) are included, the complexity of calculating self-realization increases by a sum of cubes factor.

The number of loops to realize all the transfer functions generated by the model in Figure 18 is calculated by ($n =$ number of partners):

$$L(n) = \sum_{i=1}^{n} \left( \frac{n}{(n-i)} \right)^3$$

(13)

For a dualism this creates 12 loops. For four partners the number of loops quickly grows to 8,992. Without deriving all the loops this emphasizes the realization that these models become complex very quickly as human characteristics and interactions are accounted for.

Reducing the number of bilateral channels can reduced the growth of control loops. When the conscious channels are eliminated and only the unconscious coupling is used the calculation for the number of loops becomes:

$$S(n) = n + n \sum_{i=1}^{n} \frac{1}{(i-1)(n+1-i)}; \quad n \geq 2, \left( \frac{1}{0!} = 1 \right)$$

(14)

For a dualism there are only 3 loops; for four units, 24. As more partners join in interaction, the number of loops to evaluate grows much more slowly than in Equation 13.

**Disturbances in the Dualism**

Just as disturbances affect the autonomous individual $P_1$, disturbance can also affect partners in a dualism. From Figure 8 we explore the effects of disturbances $z_1$, $u_2$, and $z_2$ on $P_1$. They are in order: direct disturbances on $P_1$, $P_2$’s goal, and disturbances on $P_2$. When each disturbance is evaluated alone with the others set to zero Starkermann shows the how their effects can be calculated. In the following formulae, the algebraic term $(1+G_1)(1+G_2)-S_{12}S_{21}G_1G_2$ provides the common denominator for all evaluations and is represented by the variable $D$. 
Common Denominator: \[ D = (1 + G_1)(1 + G_2) - S_{12}S_{21}G_1G_2 \] (15)

\[ x_1(z_1) = \pm \frac{1 + (1 - S_{12}S_{21})G_2}{D}z_1 \] (16)

\[ x_1(u_2) = \pm \frac{S_{12}}{D_2}G_2u_2 \] (17)

\[ x_1(z_2) = \pm \frac{S_{12}}{D}z_2 \] (18)

The ± indicates the disturbance can be beneficial or harmful in relation to the goal \( u_1 \).

For \( x_1(z_1) \) the results are quite distinct. Setting \( G_1=1 \) and \( G_2=4 \) we find:

\[ x_1(z_1) \text{ for devotion is } -64\% \]
\[ x_1(z_1) \text{ for hostility is } -17\% \]

The ability of a disturbance to affect a partner in amity is almost four times that in hostility. The sensitivity of \( P_1 \) to \( z_1 \) in amity is dependent upon the ratio between the two wills.

A partnership in hostility develops a myopia of hate and has a high resistance to outside influences. The hostility becomes a regenerative activity. Starkermann brings to light Hobbesian proverbs that illustrate this point.

*Bellum omnium contra omnes*; the war of all against all.

*Lupus est homo homini*; man is the wolf to man

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4 The original text finds the value as -66% and -16% by interpolation from a chart. The values shown were directly calculated.

5 Levin, Robert. *The Evil that Men Do*; book review of *The Lucifer Effect: Understanding How Good People Turn Evil*, Philip Zimbardo. Random House 2007; in American Scientist, September-October 2007. [http://www.americanscientist.org/template/BookReviewTypeDetail/assetid/55923](http://www.americanscientist.org/template/BookReviewTypeDetail/assetid/55923), retrieved 18 December 2007: In the 1971 Stanford Prison experiment conducted by Phil Zimbardo, the participants in the roles of guards and prisoners quickly aligned into a hostile interaction. Zimbardo acted in the role of warden. This role acceptance even extended to family members. Only when a woman whom Zimbardo once had a relationship with (amity) assisted on the project and pointed out the problems (disturbance in amity) did he realize how far the experiment had gone. This is a cursory view of the events and may suffer from confirmation bias. As an anecdote it aligns with Starkermann’s models. The event is fully documented at [http://www.prisonexp.org](http://www.prisonexp.org).

6 Hobbes, Thomas. *Elementa philosophica de cive* (De Cive), Amsterdam, 1647

The issues that affect $x_1(u_2)$ are the volition of $G_2$ and whether $u_2$ aligns with $u_1$ or opposes it. In general $u_2$ in amity should align or be neutral to $u_1$. To be against $u_1$ would be hypocrisy in friendship. This is a special case which is not explored at this time.

From equation 17, $P_2$’s goal is more disturbing to $P_1$ in hostility than in friendship. The effect depends on the intensity of $S_{12}$, will $G_2$, and the level of the disturbance $u_2$.

Having a stronger effect in enmity does not mean the hostility will be avoided. Enmity only has one case: $S_{12}>0; S_{21}>0$. Amity exists in two cases: $S_{12}<0; S_{21}>0$, or $S_{12}>0; S_{21}<0$. Enmity is more stable the amity and easier to model. The stability afforded by enmity may make switching to a hostile relationship preferred despite the disturbance. Starkermann makes use of proverbs to illustrate the issue:

*Mars gravior sub pace latet;* A more serious war hides under the guise of peace

*Si vi pacem, para bellum;* If you want peace, prepare for war

How does the disturbance ($z_2$) of partner $P_2$ relate to $P_1$? It mostly depends on the denominator factor $D$ which in turn is affected by the coupling factor $S_{12}S_{21}$. The result is $P_2$ disturbance has greater effect on $P_1$ in hostility than amity. The degree is less than $u_2$ since in this case $G_2$ is not a multiplying factor in the numerator.

### Numerical Examples

With all the elements, blocks, and signals defined, numerical examples will help bring perspective to their meaning. There are three main examples to study found by examination of the stability boundaries of the Starkermann resonance. They are – $\alpha$: $G_1$ and $G_2$ are in parity; $\beta$: $G_1$ is much less than $G_2$; $\gamma$: $G_1$ is much greater than $G_2$. Table 2 lists goal attainments of $x_1$ for $u_1$, $\pm u_2$, $z_1$, $z_2$ and the sums of $x_1(u_1)+x_1(\pm u_2)$. All goals ($u_n$) and disturbances ($z_n$) are evaluated at 100%. $S_{12}$ and $S_{21}$ are represented by the signs “+” or “-” and represent the attitude of the partnership. The case of (– –) is shown only for completeness of the data set.

<table>
<thead>
<tr>
<th>Goal Attainment</th>
<th>$\alpha$ ($G_1=G_2$)</th>
<th>$\beta$ ($G_1&lt;&lt;G_2$)</th>
<th>$\gamma$ ($G_1&gt;&gt;G_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Enmity</td>
<td>Amity</td>
<td>Enmity</td>
</tr>
<tr>
<td>$x_1(u_1)$</td>
<td>+ +</td>
<td>+ +</td>
<td>+ +</td>
</tr>
<tr>
<td>$x_1(z_1)$</td>
<td>11</td>
<td>11</td>
<td>38</td>
</tr>
<tr>
<td>$x_1(u_2=+1)$</td>
<td>44</td>
<td>44</td>
<td>77</td>
</tr>
<tr>
<td>$x_1(z_2)$</td>
<td>11</td>
<td>-11</td>
<td>8</td>
</tr>
<tr>
<td>$x_1(u_2=-1)$</td>
<td>-44</td>
<td>44</td>
<td>15</td>
</tr>
<tr>
<td>$x_1(u_1)+x_1(u_2=+1)$</td>
<td>89</td>
<td>0</td>
<td>92</td>
</tr>
<tr>
<td>$x_1(u_1)+x_1(u_2=-1)$</td>
<td>0</td>
<td>89</td>
<td>62</td>
</tr>
</tbody>
</table>

Table 2 – Numerical Values in Dualism
There are many more permutations that can be explored once other factors are defined to account for madness, pathology, sacrifice, and other uncountable combinations. The basic fact is that there is no average behavior, only approximations. There is also no set limit for a goal attainment necessary for survival. However negative values represent being killed or destroyed. Another thing to consider is that shifting from (+ – ) or ( – +) to (++) or the reverse can have drastic effects on $P_1$’s survival.

The values for $x_1(u_1)+x_1(u_2=\pm1)$ represent partnerships with interrelated goals and provide the basis for a simple examination. We will look at a weak $P_1$.

The absolute worse situation for a weak $P_1$ is $\beta 7$ with a goal attainment of -67%. It represents $P_1$ with will $G_1=1$ being damaged in a hostile situation by a strong $P_2$ ($G_2=7$). The weak willed partner is always destroyed in enmity. In the reverse case in $\gamma 7$ of $P_1$ with a strong will ($G_1=7$ vs. $G_2=1$) goal attainment becomes +67% and $P_1$ survives.

The best case for a weak $P_1$ is shown in $\beta 6$. Being in amity and aligned goals, with $P_2$ providing the devotion, gives $P_1$ a success rate of 97%. The danger is that a strong-willed partner providing devotion ($S_{21} = -1$) may believe they are being exploited. $P_2$ can make changes to stop the exploitation:

1) Become aggressive and change $S_{21}$ from - to +. $P_1$ goes from 97% to 89%
2) Stay aggressive and make the goals incompatible so that attainment becomes $x_1(u_1)$. $P_1$’s attainment is shifted to $\beta 1$ at 11%
3) Turn aggressive and make the goals antagonistic. This is situation $\beta 7$ and $P_1$ is ruined at -67%.

Table 2 indicates the best situation for $P_1$ is to be in amity with either aligned goals ($u_2=+1$), the stronger will, or both. Goal attainment of 97% is realized in this case. Other conditions including the impact of disturbances $z_n$ are explained in the full volume and left to the reader to explore.

Starkermann again relates the wisdom of proverbs to the situation:

A bad peace is better than a good war.
If you can’t beat them, join them.

The results resolve to a seeming paradox; since friendship is the most beneficial relationship for long-term self-realization, why then are hate and war so attractive?

Based on the models and rules explored Starkermann proposes these reasons:

a) Hostile systems at parity of willpower allow twice the will to be expressed before becoming unstable.
b) Hostile systems can act twice as fast as consensual systems; Time is the currency of survival.
c) Hostile systems have a larger range of stability than consensual systems. Stability in nature indicates survivability.

d) A victor can take from his opponent using that to increase his own chances of survival.

The concepts of will and speed for survival extends from blades of grass to ferocious animals. A dead animal has less speed and will than the worms or plants that live off it. They in turn have their own predators. What becomes evident is that even with multiple units, hostility and autonomy do not lose speed of action. Consensual units become increasingly slower in their ability to act as the number of connected members grows.

Starkermann provides another thought: Besides a dualism of correlated goals, a situation could exist of a common goal and common feedback. Here the goal $u_k$ is set by a higher authority. A conditions is created where not a single unit has a possibility or right to define his own desires, or even be autonomous. This defines totalitarianism and slavery.

4 – Variations on a Theme

The basic model is established and general behavioral patterns for different levels of will have been demonstrated. Until now the units have had parity of speed. The function block $F_1$, defining speed of action for will, has been kept the same across all patterns. Units acting at different speeds will be explored next. Also for consideration is the term for will which to this point has been 1-dimensional. It has only acted on an error signal without taking into account the rate of change or the resistance to change. Those characteristics of anticipation and enforcement (or perseverance) will also be examined.

Individual of Different Patterns of Motion

Previously $F_1$ has been held to a value of 1. This has made analysis simpler and accessible. That $F_1$ can vary has been implied in the discussions on stability. $F_1$ represent the speed at which force or will can be applied. It can be a thought, action, or process; anything that takes time.

Table 3 lists different ratios of $F_1$ to $F_2$ in a relationship. Higher values indicate faster speed. Charts for consensual (Figure 19) and hostile (Figure 20) interactions for the four ratios are shown. Colors have been traced over the resonance boundaries in Figure 19 to make the chart more readable.

<table>
<thead>
<tr>
<th>Example</th>
<th>$F_1$</th>
<th>$F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1.25</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3 - Relative Velocities of $F_1$ and $F_2$
Line A represents the partners with equal speed. Line D shows a high speed ratio with \( P_1 \) being much faster than \( P_2 \). We can inspect the effects of \( F_1/F_2 \) on the partners by using the examples from curve C for each relationship. The charts are redrawn for clarity:

In consensual partnerships we have seen the high degree of dependency between wills, goals and disturbances. This dependency extends to the speed of actions. In Figure 21 if \( P_2 \) is satisfied with a will of 2.5 then \( P_1 \) can extend his will to approximately 9 and an overall speed for the partnership of 2.06. However if \( P_2 \) wishes to increase his will to 6, then \( P_1 \) being faster and more flexible must lower his will to .3. The speed of the dualism in this situation is approximately .86. To maintain the friendship \( P_1 \) has to reduce will and speed to far below what he could achieve alone in autonomy (will 8, speed 2).
In this situation where the sluggish partner dominates with greater *will*, i.e. P1 is not able to exert *will* or speed, the partnership will dissolve into autonomy or hostility.

For the hostile relation, the stubborn P2 can dominate and achieve a *will* of 11, greater than autonomy if P1 reduces his *will*. The speed of this dualism for G2>G1 is greater than that of the consentient one and the area of stability is larger. Hostility is again shown to be agile and stable.

One last point is that very high speed ratios may be unlikely. From the truncated chart of Figure 20, when F1/F2=5 then P1 can exert a near autonomous *will* of ≈ 8 while P2’s *will* grows. G1 only becomes 4 when G2 is 110. The slow aggressor can dominate with his stubborn *will*.

**Anticipation and Enforcement**

Intelligence has been defined as the *will* G working through the pattern of motion F. The value for G has been a simple gain value up to this point. There are two qualities that can also be taken into account for G. We call them anticipation and enforcement.

Anticipation allows an individual to increase *will* quickly through foresight or preconception of events. Anticipation is modeled as the derivative of the error signal ε(t), dε(t)/dt. Its Laplace form is Ca:s: “a” represents anticipation. Anticipation speeds up goal approach but not goal attainment (x_n).

Enforcement of the will is modeled as the integral of the error signal. It is represented as ∫ ε(t)dt and its Laplace form is Cp/s: “p” represents perseverance. Enforcement (perseverance) increases *will* slowly over time by taking into account the past. It slows down goal approach but increases final goal attainment.

The model in Figure 23 shows a situation where one individual’s *will* includes anticipation and the other includes enforcement. Implied is that each unit does have anticipation and enforcement factors. In this particular example C_p1=0; C_a2=0.

P1 and P2 will be the same fast and slow units from the last section. This allows comparison of anticipation and enforcement factors to familiar models.
First order effects are examined for the case of $P_1$ having mild anticipation ($C_{a1}=0.2$). There is no enforcement for $P_2$ ($C_{p2}/s=0$). Figure 24 illustrates that in amity $P_1$ has an opportunity to extend his will higher than before only if $P_2$ keeps his will below approximately 2.5. If $P_2$ retains a high level of will $P_1$ is no better off than before. This again shows that a stubborn and slower partner in a friendship defines its stability. If $P_2$ is too stubborn the friendship may not last.

Adding anticipation to a hostile relation shows both parties are able to increase their will. $P_2$ can extend $G_1$ up to 16, almost twice as before. At that point $P_1$ is 4, much better than 2.5 from the previous section. Anticipation and the increase it brings in speed provides the opportunity for extension of will and a larger area of stability. Once more Nature appears to favor hostility.

We look at adding an enforcement factor for $P_2$ of $C_{p2}=0.2$. $P_2$’s increased stubbornness in the consensual relationship of Figure 26 further limits $P_1$ if $P_2$ opts to increase his will. At $G_2=4$, $P_1$ has almost no ability to act. By adding enforcement $P_2$ has also reduced his own range from 8 to 5. The close up view of this is shown in Figure 27. The effect of adding enforcement in amity increases the possibility of instability.

Anticipation and enforcement for a hostile relationship are shown in Figure 28. The graph of amity is overlaid for comparison. This stable region is almost identical to the previous chart for hostility. Both enforcement and anticipation increase the stability in a hostile relationship.

Tables similar to Table 2 can be compiled to show numeric relations of goal alignment to anticipatory and enforcing modes. To maintain a semblance of brevity the reader is asked to consult the book for an example.
Figure 26 – Consent: anticipation and enforcement

Figure 27 – Consent: anticipation and enforcement; closer view

Figure 28 - Hostility: anticipation and enforcement; amity in comparison
5 – From One to Many

The process of modeling multiple interconnected partners and aggression in groups extends from the work performed on the dualistic model. These configurations are the next areas to explore.

**Multi-Partner Systems**

The first rule is that all units will either be in enmity or amity. There is no mixing of hostility and friendship in a single system. The intensity of $S_{12}$ and $S_{21}$ can vary from $\pm 0.5$ to $\pm 1.0$. This makes the coupling factor range from $\pm 0.25$ to $\pm 1.0$. A four unit model with the unconscious relations is shown in Figure 29 with 24 loops of flow in the unconscious interactions. For comparison 8 units would have 16,072 loops to contemplate.

A system like this can consist of relatives, business partners, business competitors, etc. There are four factors that are of interest in evaluating this kind of system.

A) The maximum will $G_i$ each partner can express in this kind of system
B) The speed of action, $v$, of the system.
C) The goal attainment $x_i/u_i$.
D) The effect of a disturbance $x_i/z_i$ before the system collapses into instability.

This is for consentient and hostile configurations.

The quantitative domains for each of these four factors are again best illustrated using charts.
For factor A the maximum level of will $G_i$ is strongly related to the number of units involved. Friendship (+ -) requires a greater decrease in $G_i$ than hostility (+ +). Notice the huge reduction in $G_i$ for (+ -)1 from the autonomous will of 8. Overall $G_i$ for hostility is still about twice that as for amity compared to relative strengths of the coupling factors.

This drastic decrease in expressible will indicates that individual goals and large groups do not work well together.

In factor B we find a velocity for action decreases for a friendly group but does not go to 0 as $n$ goes to $\infty$. Instead it reaches a limit of about 10. The hostile groups, just as in a dualism, see no decrease in velocity according to $N$ but only for friendly.

It seems that large groups with many different goals can still act even though goal attainment and will approach zero. Starkermann provides large bureaucracies as prime exhibits of this type of behavior: *Strenua inertia!* (Energetic idleness, Horace)

The telling item for factors A and B is for the partner providing devotion $S_{21}$ is negative in devotion. Becoming hostile and changing $S_{21}$ to a positive value will provide a huge increase in will and speed of action.
Even though the two previous factors promise hostility will bring greater speed and level of will, survival depends on achieving goals. Factor C, goal attainment, like prior factors A and B, decreases with increasing interacting units. Unlike them, goal attainment for strong enmity drops precipitously compared to all other relations. Casual friendship (+-)0.5 and mild disdain (++)0.5 decrease the least. Normal friendship (+)1.0 maintains a large advantage over normal hate (++)1.0. This indicates that goal attainment is helped by amity over the long term.

Enmity is attractive in the here and now but is damaging over time. And since enmity is so attractive and mutually damaging the situation again calls on the Latin phrase, *bellum omnium contra omnes* – war of all against all.

As shown in the model for a dualism, friendship is highly influenced by disturbances. Factor D shows that this influence increases for all relationships except for normal enmity. Normal enmity has a constant level; beyond that it ignores disturbances. Normal friendship is most susceptible to outside disturbances with the effect being greater than 100% of the desired goal indicating a total collapse of the system.

One observation is that this shows attempts to bring peace to parties in strong enmity will be ignored. Ignored that is, until the damage is so great the antagonists have no power or will to carry on their fight.
The summary findings of the model in Figure 29 show a stark contrast between amity and enmity. Hostility strives to increase will ruthlessly, creates heavy mutual damage, is quick to act, and is imperturbable to distractions such as treaty negotiations. Consent in turn surrenders its will, provides some mutual help, is slow and sluggish to act, and is highly vulnerable to outside distractions.

**The Group Aggression Phenomenon**

There is one more model to consider, that of groups interacting with each other. This can be shown as an extension of the bilateral dualism from Section 3. Figure 34 shows the main construct with groups $G_a$ and $G_b$ interacting. Each group has two units represented by $P_i$. A distinction between this configuration and a multi-partner system is that the groups interact only via the conscious observation signal $V_i$. Just in this diagram there are over 30 parameters to be evaluated so simplification is necessary to even attempt an explanation.

![Figure 34 - Bilateral Group Interaction](image)

Except where noted block elements have the same functions as previously defined. Key elements of the groups and their interaction to note are:

- Each partner $P$ has his own goal $u_i$ and goal attainment $x_i/u_i$.
- Group goal attainment $y_a$ is the sum of goal attainment for the partners. i.e. united strength.
- The groups interact by mutual observation via the conscious transfer function $V_i$. $V_i$ can be imaginary or real and may be greater than 1. Factors for $V_i$ are called Aggression which will be designated with an “A”. The strength of aggression will be the positive cross-feed signal of $V$ between groups.
- Unconscious perception of group members and their membership is via the functions $S_{12}$, $S_{21}$, $S_{34}$, and $S_{43}$. This is also called the group attitude. Within
each group the cross-feed will be devotional with a negative sign indicating everyone is working together. The cross-feed function will be designated “D”.

- **Will**, $G$, for a group has a similar time delay factor as $F_i$ does for an individual.

Other criteria exist for this model to be evaluated:

- Each individual is responsible for his own goal.
- Individuals and groups can become unstable.
- Bilateral group hostility is nourished by mutual observation of the other group’s actions.
- Internal group dynamics can be indifferent, competitive, or supportive. For the case presented they will be supportive.
- Members do not know the overall group goal attainment. They focus on their own goals and assume being loyal to the group justifies behavior. This loyalty is enforced by observing the other groups attainment. In other words, there is no overriding group goal.

The model configured for calculations in Figure 35 shows the Devotional (D) and Aggressive transfer factors and the groups’ *wills* as Laplace transfers. **D** and **A** can either act immediately or with a delay. This is indicated by $m$ or $p > 0$ for delay and $m$ or $p = 0$ for immediacy of action. The will block **G** can also have a time delay represented by $n$. The **D** and **A** terms are the primary operators for analyzing group interactions.

![Figure 35 - Bilateral groups with transfer functions](image-url)
There are four formulae to consider in evaluation group interactions. The better understanding of these formulae, as with all the previous material, will come from explaining and visualizing the data.

Steady state goal attainment of $y_a(u_a)$:

$$y_a(u_a) = \frac{2G[(1 + G)^2 + D^2G^2[I + G(I + GD)]^2]}{[(1 + G)^2 + D^2G^2]^2 - 4A^2G^2[I + G(I + GD)^2]^2}$$  \hspace{1cm} (19)$$

Disturbance of one group's goal on the other when they are opposite and antagonistic: $u_a = -u_b$.

$$y_a(u_b) = \frac{4AG^2[I + G(I + D)]^2}{[(1 + G)^2 + D^2G^2]^2 - 4A^2G^2[I + G(I + GD)^2]^2}$$ \hspace{1cm} (20)$$

Effect of disturbance $y_a(u_b)$ on $y_a(u_a)$: $y_a(u_a) - y_a(u_b)$

$$y_a(u_a) - y_a(u_b) = \frac{1 + G(I + D^2)}{(1 + G)^2 + D^2G^2 + 2AG[I + G(I + GD)^2]^2} \times 2G$$ \hspace{1cm} (21)$$

When $G$ becomes very large (i.e. $\infty$), Equation 21 becomes

$$y_a(u_a) - y_a(u_b) \bigg|_{G=\infty} = \frac{2}{1 + 2A}$$ \hspace{1cm} (22)$$

A quick analysis provides us this information:

Assuming all $G$'s are equal, extremely high levels of aggression ($A$) will ruin goal attainment for both groups.

For $G>0$ (positive wills), $y_a(u_a) - y_a(u_b)$ increases with $D$ and decreases with $A$.

For very large $G$, $D$ becomes nonexistent and $A$ causes more destruction to both groups.

The pertinent charts to derive from this will be for goal attainment shown in Equation 19 and the effect of disturbance from the other group in Equation 21.
In the example of Figure 36 we see goal attainment $y_a(u_a)$ with respect to will $G$. Devotion has a value of 0.5 indicating some time delay. The line $A=0.5$ represents formula 22. When $A>0.5$ poles start to occur. It is not shown on this chart, but when $G$ becomes larger than the pole value for a particular $A_i$, then $y_a(u_a)$ becomes negative. In the chart, for $A=0.75$ goal attainment instantly turns negative when $G>1.75$. This represents self-destruction.

Here, Starkermann quotes Napoleon:

$Du sublime au ridicule il n'y a qu'un pas!$

There is only one step from the sublime to the ridiculous!

And suggests aggression > 0.5 leads to creating, in his words, *Napoleonic transgressions*. History is the teacher here.

Figure 36 had demonstrated the case of a group without a disturbance. This indicates it is aggressive but not able to engage or attack another group. A group vs. group behavior is shown in Figure 37 which shows the effect of various levels of $A$ with variations in $D$. The largest success for goals is when aggression is least and devotion is high ($A=0, D=1$). This Note that at $A=2, D=0$ and goals are less than 0.4. High aggression again leads to increased destruction of the group.
The last thing to consider with groups is the time delay element which gives rise to stability. Homeostasis of the individual and partnerships have been considered and mapped. The same is possible for the group interaction.

The time elements for groups are the delays for devotion and aggression. They in turn define the limits of stability. In Figure 38 to the right, Devotion acts instantly and Aggression has a slight delay. The chart uses the European format of a comma in place of a decimal. The level of group members’ will G allowed for stability is shown by the isomorphic lines. It follows that as individual will decreases, the amount of allowable Devotion and Aggression for the group increases. In other words group Devotion and Aggression to the enemy has more value than self-realization.

As Starkermann notes, the term delayed aggression relates to the preparation for action. The preparation for war is considered delayed aggression.

Looking at the opposite case of delayed/slow Devotion and instant Aggression gives us the shape in Figure 39. The stability region allows Devotion to remain large due to the fast Aggression. Fast Aggression also permits a wider area for G, will for self-realization, to act.

The group with fast Aggression is more intelligent using the term defined earlier that intelligence is the ability to reach a goal.

As shown previously at high values for A, the term D has no influence and the group suffers damage due to the increased violence. Numeric extrapolations of the values for goal attainment for these examples are found in the original text.
The last example of group interaction compares groups with equal levels of will but one group’s will acts faster than the other. This is related to the exponent “n” in the Laplace form for G from Figure 35. Ga=Gb=4 but n(Ga)=2 and n(Gb)=3.

The stability domain for this chart shows that Devotion increases sharply for much of the range of Aggression. The fast moving group anticipates joy of victory. The slower moving group endures the agony of defeat. Both situations stimulate the growth of Devotion in the respective groups. As has been seen before, the higher stages of Aggression force Devotion to the side and everything coalesces around the ongoing fight and the mutual damage it causes.

**Summarizing Group Aggression**

Individuals, weak-willed in autonomy and self-realization, tend to forms groups that in turn become aggressive towards other groups. The members develop high devotion to the group because of power brought by the strength in numbers. The lower the will G, which here can be viewed as an indication of self-value or self-image, the higher the group devotion. As the devotion increases so does the aggression factor.

When engaged with another group, overzealous aggression causes mutual destruction to both groups in the case of equal will. This is derived from the studies of the individual and dualism. The message is the same; the strong devour the weak.

Individuals with high self-image, i.e. high internal G are not suited for large groups. Groups with high G-value members will fragment. On the other hand those who cannot or have not developed their will to become independent have to join together and fight.

Starkermann points out one last element for this exercise in groups. It casts a light on the incredible complexity this type of analysis generates with even a few factors. He points out that for Figure 34 there are forty-two loops to be calculated for each iteration of the model. Adding factors or units/groups magnifies the complexity. The point he makes is that a person cannot fully comprehend the system of social connections to which anyone belongs.
6 – Through the Looking Glass

The Loop Structure of the Mammalian Brain

The exploration of the actions of people and groups has been based on applying control systems structure to the group dynamics. It turns out that the neural structure of mammalian brains is also similar to technical control systems. This complexity of this system bears exploring.

The neurons with their synapses, axons, and dendrites manipulate a large amount of information and connection. The neural network in Figure 41 with it’s looping of information is analogous to the organizational unit that have been studied. It is also not so far from the structure of the control loop of Figure 42. The extension of a control system to neurons can be modeled as an operational block. Information comes into one operational block where it is acted on. One output channel, the axon sends the information out through dendrites. A three neuron system can be drawn comprising five signal loops: three with two operational blocks and two with three operational blocks. A four-neuron system grows to twenty loops.

As a neural system grows the number of connections increases as:

\[ L(n) = n! \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{1}{k!(n-k)}; \quad n \geq 2 \]  

(23)
The twelve neuron diagram of Figure 45 represents 119,481,282 loops. A human brain with $10^{15}$ neurons would not have total connectivity between each neuron. Such total information connectivity would lead to chaos.

It can be allowed that neurons would have different control and transfer functions and a variety of connections. Because of the huge number of neurons the permutations are practically incalculable. The distinction is that due to this numerical complexity the uniqueness of each individual person is assured.

**Figure 45 - Twelve interconnected neurons**

### Closing remarks
Exploring dualism brought to light that many aspects of social behavior can be estimated from the basic model. A definition of attitude was found by looking at the unconscious bilateral interchange of $S_{12}\delta_2$ and $S_{21}\delta_1$. Later the observational attitude $V$ was explored in group interactions.

The critical aspect of interrelations is the structure of amity. Amity indicates one partner (for example: $P_1$) creates the devotional attitude by adding a negative sign to the incoming unconscious stream making $-S_{12}\delta_2$. By merely changing his mind, the attitude changes from $(-+)$ and becomes $(++)$. At that point amity switches to enmity.

Proverbs come into play here as well:

- *The closest friends, the worst enemies.*
- *Better and open enemy than a false friend.*

Starkermann proposes a few areas of further exploration.

The study could be expanded by considering a case where a person could anticipate the attitude relation. In other words, the person intuitively understands the unconscious perspective of the other person. This leads to a model where each person has two brains, one conscious and the other unconscious. At this level however, the complexity has increased from 3 to 20 loops.

The magnitude of the feedback signal $R_1$ could be varied to reflect the amount of awareness or (in)sensitivity to achievement a person possesses.
The key realization is that modeling human behavior mathematically requires great care with the analogies presented. The work is difficult and made more so in order to find analogies that work for an interdisciplinary study. Scientists will defend invasions into their territory from outsiders: this is natural behavior. However defining the quantitative laws for nature requires crossing disciplinary boundaries. The attempt to enter the qualitative socio-psychological domain with mathematical formulizations is tantamount to crossing the Rubicon.

Attempting to transfer social situations to mathematical models is difficult and even the most simple models create a large amount of work to understand. The use of simple linear elements can create a large variety of social behaviors. The model built is rational according to the laws of Nature. Adding new components helps discover new facts while magnifying complexity. Just as in nature, simple building blocks come together to create incomprehensible complexity.

The present description of how the world works with respect to people and events is qualitative. It draws from philosophy, psychology, sociology, and religion using verbal or pictorial explanations. The complexity of reality gives rise to the different doctrines in all areas that attempt to explain reality.

The essence though is that everything derives from natural laws of the universe and these can be applied to behavior. Care must be made in making the model to stay with limits of what is verifiable. There are probably more than five human senses related to human interaction. The unconscious perception of attitude is one that has been explicitly used.

There is no attempt to deny or create religion. The world is so complex that symbolism and mythology are necessary for people to find an explanation for the world and life around them. In the finding of a few social facts the huge effort required to further discover natural laws has become self evident.

Starkermann’s penchant for quotes and proverbs ends the volume with these words from Bertrand Russell:

> What was most remote from ourselves was first brought under the domain of law, and then gradually; what was nearer: first the heavens, next the earth, then animal and vegetable life, then the human body; and last of all (as yet imperfectly) the human mind.  

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7 – Appendices

The appendices to this work provide greater detail than time or sanity allow to explore. A brief listing of the contents will help explain the depth Dr. Starkermann went to insure completeness of his study.

Appendix I
Provide the general formula for goal attainment of Figure V-1 in the book. (Figure 8 here)

Appendix II
Develops the feedback transfer function for the book Figure V-9 (Figure 18)

Appendix III
Full develops the argument and control system examples to define the unconscious and the time dependency of the models.

Appendix IV
Develops in greater detail the aspects of Anticipation and Enforcement with respect to self-realization. Shows how block G is actually modeled as a Proportional-Integral-Derivative (PID) controller.

Appendix V
Shows the characteristic equation for the structure for Figure IX-1 (Figure 34)